

# APPLICATION OF STEP DRAWDOWN PUMPING TESTS IN CONSOLIDATED ROCK AQUIFERS

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## Abstract

The concept of step-drawdown pumping tests was first introduced by Jacob (1947), and modifications in step-test analytical methods were subsequently developed by Rorabaugh (1953) and others. The analysis of step-drawdown pumping test data enables the quantification of the components of drawdown in a pumped well due to: **a.** formation or aquifer loss and **b.** well loss.

This paper summarizes the results from hundreds of step-drawdown pumping tests run on production wells that have been completed in crystalline, igneous and sedimentary rock aquifers in India and in sedimentary rock aquifers in the northeastern United States. The test data were analyzed by a graphical technique (Bruin and Hudson, 1955) and by Rorabaugh's method (1953).

The test analyses indicate that well losses in bedrock wells often comprise a significant portion (often greater than 50 percent) of the total drawdown in a pumped well and should be taken into account when developing estimates of the safe sustained yield for individual bedrock wells. Anomalies in the step-drawdown pumping test data are useful for interpreting aquifer conditions such as fracture dewatering, leakage, and negative boundary conditions.

The general body of data derived from the tests analyses also indicates that both the well loss constant "C" and aquifer loss constant "B" generally decrease with an increase in specific capacity. A significant decrease in specific capacity during a step test is common in wells showing high well losses. The nature of well loss in consolidated rock aquifers is discussed and a number of practical applications of step-drawdown pumping tests are outlined.

## Introduction

The principal objectives of this paper are to:

- Present the results of step-drawdown pumping tests conducted on production wells completed in a variety of consolidated rock aquifers.
- Quantify the nature and magnitude of well loss in production wells completed in consolidated rock aquifers.
- Discuss practical applications of step-drawdown pumping tests.

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## Pumping Test Procedures and Test Analysis

Most of the wells tested were installed using the air-hammer drilling method and well diameters ranged from 4 to 8 inches. Submersible pumps were generally used for testing and the discharge pipes were equipped with totalizing water meters and/or a calibrated orifice for flow measurement. Water levels were measured in the pumping wells using electronic sounding equipment or transducers. The step-drawdown pumping tests were generally a precursor to longer term constant-rate pumping tests and the data derived from the step tests were used to estimate a sustained yield for the longer term constant-rate tests.

The test data were analyzed by a graphical solution to Jacob's equation developed by Bruin and Hudson (1955). The basic theory of step-drawdown test analysis is provided below.

Jacob (1947) introduced the concept of a multiple step-drawdown pumping test with the objective of determining well losses and the effective radius of a well. Jacob noted that drawdown in a pumping well has two components: the first component termed "Aquifer or Formation Loss" arises from the "resistance" of the aquifer matrix to fluid flow. Aquifer loss is proportional to discharge (Q) and increases with time as the cone of influence expands. The second component, termed "Well Loss", represents the loss of head that accompanies the flow through a well screen (or water bearing fractures in an open-hole well) and in the casing. Well loss is proportional to the square of the discharge (Q) and is independent of time. Jacob defined the following equation:

$$s_w = BQ + CQ^2 \quad (1)$$

Where:

- $s_w$ : Total drawdown in a pumping well [L].
- Q: Pumping Rate [ $L^3/t$ ].
- BQ: Component of drawdown due to aquifer or formation loss [L].
- $CQ^2$ : Component of drawdown due to well loss [L].
- B: Aquifer loss constant [ $t/L^2$ ]. B represents the total resistance of the aquifer matrix from the well wall out to the radius of influence.
- C: Well loss constant [ $t^2/L^5$ ].

Another way of expressing the total drawdown in a pumping well is:

$$s_w = Q/(4\pi T) * \{\ln(4tT/r_w^2 S) - 0.5772\} + CQ^2 \quad (2)$$

Where:

- B:  $1/4\pi T * \{\ln(4tT/r_w^2 S) - 0.5772\}$
- T: Transmissivity [ $L^2/t$ ].
- t: Time [t].
- S: Storativity (dimensionless).
- $r_w$ : Effective well radius [L].

Bruin and Hudson (1955) presented a graphical solution for the equation:  $s_w = BQ + CQ^2$  which can be used to determine the aquifer and well loss constants B and C which is demonstrated in the example below.

### Step-Drawdown Pumping Test Example

A step-drawdown pumping test was conducted on an 8-inch diameter test well completed in siltstones and shales of the Triassic Passaic Formation in Central New Jersey. The total depth of the well is 400 feet with an open-hole interval of 320 feet. The step-drawdown test involved pumping the well at four successively increasing pumping rates or steps each of 0.5-hour duration. The initial pumping rate was 190 gallons per minute (gpm) followed by 300 gpm, 398 gpm, and 455 gpm. The data developed from this test were analyzed to determine the magnitude of drawdown due to aquifer or formation loss (BQ) and well loss ( $CQ^2$ ) and a sustainable pumping rate for a constant-rate aquifer pumping test.

The test data were analyzed by the Bruin and Hudson (1955) graphical method where the observed drawdown in the production well ( $s_w$ ) was plotted against the corresponding time (t) for each pumping step on semi-logarithmic graph paper (**Figure 1**). The curve through the plotted data for each step was then extrapolated to the end of the next step in the test. The incremental drawdown for each step ( $\Delta s$ ) is the difference between the drawdown at the end of a given step and the extrapolated drawdown from the preceding step as shown on **Figure 1**. The total drawdown ( $s_w$ ) is the sum of the incremental drawdowns. The ratio  $s_w/Q_n$  for each individual step (n) was then calculated and the data derived from the test are summarized in the table below.

**Total Drawdown ( $s_w$ ) for each Step and Specific Capacity Values ( $Q/s_w$ )**

Step	Q (gpm)	$\Delta s$ (ft)	$s_w$ (ft)	$Q/s_w$ (gpm/ft)	$s_w/Q$ (ft/gpm)
1	190	20.35	20.35	9.34	0.107
2	301	13.34	33.69	8.94	0.112
3	398	11.53	45.22	8.80	0.114
4	455	5.47	50.69	8.98	0.111

The calculated values of  $s_w/Q_n$  versus the corresponding values of  $Q_n$  were then plotted on arithmetic graph paper (**Figure 2**). This data plot generally yields a straight line with the slope equal to C (well loss constant) and the intercept with the vertical axis equal to B (aquifer loss constant). Values of  $C = 2.9 \times 10^{-5}$  ft/gpm<sup>2</sup> and  $B = 0.102$  ft/gpm were derived from the analysis.

The arithmetic plot of  $s_w/Q_n$  versus  $Q_n$  for this particular test indicated that the data from the step-drawdown test “falls off” the straight line plot between the 3<sup>rd</sup> and 4<sup>th</sup> steps and the slope of

the plot decreases. This behavior indicates leakage to the bedrock aquifer system from overlying saturated unconsolidated glacial deposits.

The resulting aquifer loss (BQ) and well loss ( $CQ^2$ ) components of drawdown are summarized below for the 4 pumping steps. The analysis indicates that well loss is a minor component of water-level drawdown in this well.

**Drawdown Components of Aquifer Loss (BQ) and Well Loss ( $CQ^2$ )**

Step	Q (gpm)	BQ (ft)	$CQ^2$ (ft)	BQ + $CQ^2$ (ft)
1	190	19.38	1.05	20.43
2	301	30.70	2.63	33.33
3	398	40.60	4.60	45.20
4	455	46.41	6.01	52.42

### Summary Discussion

The step-drawdown pumping test data for several hundred wells in consolidated rock aquifers were analyzed by the graphical solution of the equation  $s_w = BQ + CQ^2$  (Bruin and Hudson, 1955). The graphical method of plotting  $s_w/Q$  versus Q often enables the detection of boundary conditions from a change in the slope of the plot. The first plot on **Figure 3** (Test No. 102) indicates fracture dewatering after the third step; the second plot (Test No. 31) indicates leakage (recharge), and the third plot (Test No. 77) indicates a limited aquifer or negative boundary condition.

The data derived from the step-drawdown pumping tests were used to calculate an optimal pumping rate which would not result in fracture dewatering and concomitant drawdown instability for constant-rate aquifer pumping tests. When fracture dewatering occurs, the hydraulic characteristics of the aquifer in the immediate vicinity of the well bore are changed in that the number of fracture openings contributing water to a well is decreased. This results in increased non-Darcian flow and an increase in the value of the well loss coefficient (C) and for a linear plot of  $s_w/Q$  versus Q, fracture dewatering results in an increase of the slope. An aquifer of limited extent (negative boundary) will have a similar effect on the plot of  $s_w/Q$  vs. Q, however the change in slope generally will not be as pronounced as for fracture dewatering conditions. If leakage is encountered during a test, the slope of the plot will decrease.

**Tables 1, 2, and 3** present the results of step-drawdown pumping tests conducted on wells completed in crystalline, basalt, and sedimentary rock aquifer systems in India and in sedimentary rock aquifers in the Northeastern United States. In these tables, column 2 is the pumping rate for the last step of the test, column 3 provides the specific capacity for the last step, columns 4 and 5 contain the aquifer and well loss constants derived from the graphical solution; columns 6 and 7 provide the components of drawdown as a result of aquifer and well

loss computed for the maximum test discharge. Transmissivity (T), determined from constant-rate pumping tests, is provided in column 8.

A review of the step-drawdown pumping test results for the various consolidated rock aquifers indicates that:

1. Well losses comprise a considerable percentage of the total drawdown in many of the wells tested.
2. The nature of well loss in these open-hole consolidated rock wells is considered to be a result of non-Darcian flow in the aquifer in the vicinity of the pumped well.
3. The percent reduction in specific capacity during an individual test is, in general, higher for wells with high components of drawdown due to well loss.
4. **Figure 4**, a plot of specific capacity (Q/s max) versus the well loss coefficient "C", indicates that as specific capacity decreases the well loss constant, C, increases. Similar results have been noted by Walton (1962), Mogg (1968), and Eagon and Johe (1972) for wells completed in both unconsolidated and consolidated rock aquifer systems.
5. **Figure 5**, a plot of specific capacity (Q/s max) versus the aquifer loss constant (B), indicates a pattern of decreasing B with increasing Q/s. The aquifer loss coefficient B is proportional to  $\ln t/T$  (Equation 2) and as such should decrease with an increase in transmissivity (T).
6. There does not appear to be any relationship between transmissivity and the proportion of drawdown in a well due to well loss. There are cases of low transmissivity wells with both high and low well losses. The same was observed for wells with high transmissivities.

In summary, well losses comprise a significant component of the total drawdown in wells completed in consolidated rock aquifers. The quantification of the magnitude of this component of drawdown is an important input to the determination of a safe sustained pumping rate for a production well.

The practical applications of step-drawdown pumping tests include:

1. Selecting optimal pumping rates for longer term constant-rate aquifer pumping tests (to calculate transmissivity) that avoid fracture dewatering and concomitant water-level drawdown instability.
2. Determining the long-term safe sustained yield and practical pumping rates for production wells.
3. Determining aquifer boundary conditions.
4. Analysis of the effectiveness of production well redevelopment.
5. Testing wells for which little or no information is available to determine a range of operating yields (pumping rates) and depth(s) to major water-bearing fractures.

## References

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**Table 1: Step-Drawdown Pumping Test Results in Crystalline Rocks**

Well No.	Q max (gpm)	Q/s last step (gpm/ft)	B (ft/gpm)	C (ft/gpm <sup>2</sup> )	BQ (ft)	CQ <sup>2</sup> (ft)	Transmissivity (gpd/ft)	% Reduction in Specific Capacity
Amla-1	17.7	0.11	4.6	0.024	82.2	7.5	50	13
WDP 194	9.2	0.077	3.9	0.77	35.9	65.2	55	35
WDP 163	7.2	0.31	2.8	0.03	20	1.56	210	4.2
Amla-2	40	0.25	1.8	0.04	73.2	64	400	40
WDP 182	21.7	0.32	1.1	0.083	24.5	39	470	36
WDP 217	40	2.0	0.24	0.006	9.6	9.6	800	39
WDP 208	52	0.69	0.28	0.012	14.6	32.4	1,400	72
WDP 243	100	1.0	0.72	0.0085	72	85	2,500	46
WDP 220	47	1.3	0.21	0.0095	9.9	21	4,000	77
WDP 184	52	1.0	0.65	0.0077	28.5	20.8	4,300	40
WDP 195	196	6.0	0.10	0.00025	20.1	9.6	5,700	7.9
WDP 246	150	1.9	0.10	0.0025	15	56	6,000	33
WDP 175	40	2.1	0.15	0.008	6	12.8	6,600	82
WDP 244	150	6.0	0.10	0.00038	15.7	8.5	7,600	21
WDP 183	40	2.3	0.19	0.0060	7.52	9.66	7,900	35
WDP 188	25	0.63	0.20	0.055	5	34.4	9,500	51
WDP 196	196	11.6	0.062	0.000075	12.2	2.88	14,700	16
WDP 187	172	6.1	0.089	0.00033	15.3	9.76	12,000	22
WDP 131	42	1.55	0.31	0.008	13	14.1	18,500	35
WDP 222	84	5.4	0.075	0.0012	6.3	8.46	22,000	44
A 118	67	9.4	0.091	0.00018	6.1	0.8	22,000	2.5

*gpm - Gallons per Minute*

*gpd/ft - Gallons per Day per Foot*

*Q - Pumping Rate*

*B - Coefficient of Aquifer Loss*

*C - Coefficient of Well Loss*

*BQ - Aquifer Loss Component*

*CQ<sup>2</sup> - Well Loss Component*

**Table 2: Step-Drawdown Pumping Test Results in Basalt Formations**

Well No.	Q max (gpm)	Q/s last step (gpm/ft)	B (ft/gpm)	C (ft/gpm <sup>2</sup> )	BQ (ft)	CQ <sup>2</sup> (ft)	Transmissivity (gpd/ft)	% Reduction in Specific Capacity
WDP 189	20	0.13	2.1	0.17	42	68	350	57
WDP 168	42	0.3	1.55	0.02	65.1	35.3	700	7.3
WDP 262	40	0.57	0.4	0.0075	16	12	920	8
WDP-234	32	0.42	0.48	0.021	15.4	21.5	1,050	30
WDP 350	68	3.8	0.036	0.0022	2.44	10.17	1,600	68
A 216	16	0.57	0.26	0.047	4.16	12.03	1,650	65
WDP 173	37	0.43	0.32	0.0073	8.55	5.12	1,800	28
CP 492	42	4.7	0.17	0.00096	6.92	1.67	3,200	22
CP 153	120	6.0	0.11	0.00062	13.2	8.92	3,400	25
P 197	31	0.36	0.13	0.013	4.03	12.5	3,400	44
WDP 287	84	1.6	0.36	0.002	29.9	14.11	3,500	29
A 305	50	5	0.13	0.0015	6.6	3.75	5,700	20
WDP 323	75	1.4	0.08	0.0078	6	45	5,800	70
A 406	130	2.8	0.12	0.00071	15.6	11.99	6,200	31
WDP 320	181	5.6	0.041	0.00072	7.42	23.58	6,200	29
WDP 338	140	4.9	0.075	0.00055	10.5	10.78	6,300	58
WDP 388	70	3	0.18	0.002	12.39	9.8	6,500	36
WDP 391	110	7	0.12	0.0002	13.09	2.42	8,300	22
CP 325	117	4.5	0.06	0.0012	7.02	16.43	8,400	54
WDP 349	53	5.6	0.066	0.002	3.5	5.53	9,900	52
A 473	30	2.6	0.11	0.0086	3.3	7.74	11,500	51
P 133	99	7.3	0.061	0.00015	6.03	1.47	14,000	31
CP 181	220	8.1	0.073	0.00017	16.1	8.22	16,200	35
WDP 378	60	4.8	0.084	0.00075	5.04	2.7	22,600	24
A 306	90	11	0.029	0.0007	2.59	5.67	25,800	27
A 472	60	15.9	0.036	0.00037	2.16	1.33	35,500	30

*gpm - Gallons per Minute*

*gpd/ft - Gallons per Day per Foot*

*Q - Pumping Rate*

*B - Coefficient of Aquifer Loss*

*C - Coefficient of Well Loss*

*BQ - Aquifer Loss Component*

*CQ<sup>2</sup> - Well Loss Component*



**Table 3: Step-Drawdown Pumping Test Results in Sedimentary Rocks**

Well No.	Q max (gpm)	Q/s last step (gpm/ft)	B (ft/gpm)	C (ft/gpm <sup>2</sup> )	BQ (ft)	CQ <sup>2</sup> (ft)	Transmissivity (gpd/ft)	% Reduction in Specific Capacity
Gd-161	24	0.17	0.92	0.073	22.08	42.05	80	36
Gd-153	12.5	0.12	4.3	0.22	53.75	34.4	100	58
Gd-112	32	0.27	2.4	0.022	76.8	22.53	150	12
Gd-159	16	0.14	0.75	0.35	12	89.6	160	63
Gd-110	32	0.83	0.64	0.018	20.5	18.43	800	34
Gd-152	50	1.2	0.45	0.0045	22.5	11.2	950	19
Gd-156	25	0.49	0.105	0.019	2.6	11.87	1,900	18
TW-4	349	3.31	0.17	0.00063	60.72	76.73	4,300	34
TW-3	349	3.79	0.17	0.0002	61.1	24.36	4,500	25
PW-1	187	0.84	0.33	0.004	61.71	139.9	4,500	61
Gd-111	53	2.1	0.32	0.0028	17	7.8	4,700	26
TW-1	200	4.5	0.07	0.00071	14	28.4	7,200	40
TW-2	198	6.06	0.13	0.0002	26.43	7.84	10,600	15

*gpm - Gallons per Minute*

*gpd/ft - Gallons per Day per Foot*

*Q - Pumping Rate*

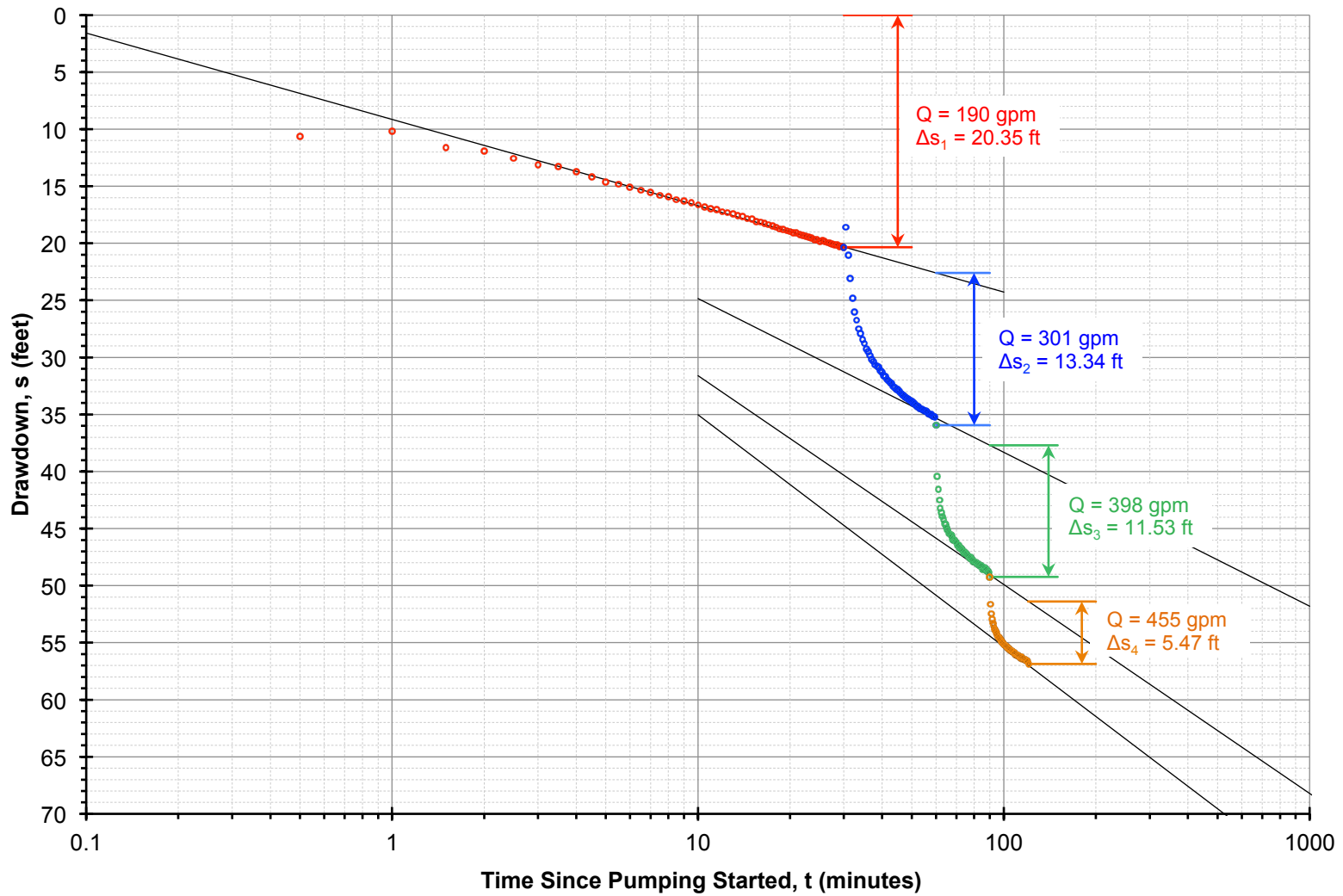
*B - Coefficient of Aquifer Loss*

*C - Coefficient of Well Loss*

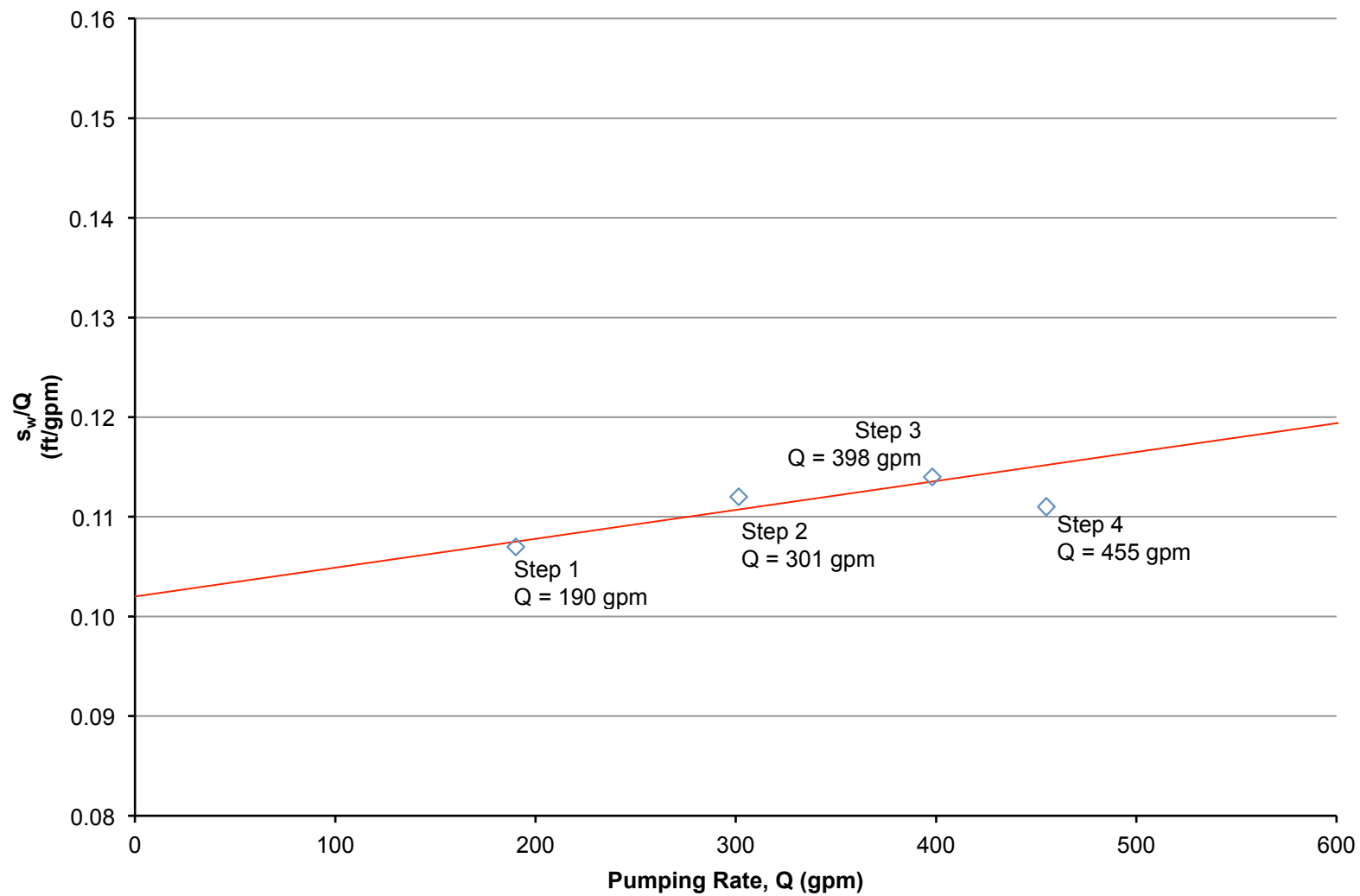
*BQ - Aquifer Loss Component*

*CQ<sup>2</sup> - Well Loss Component*

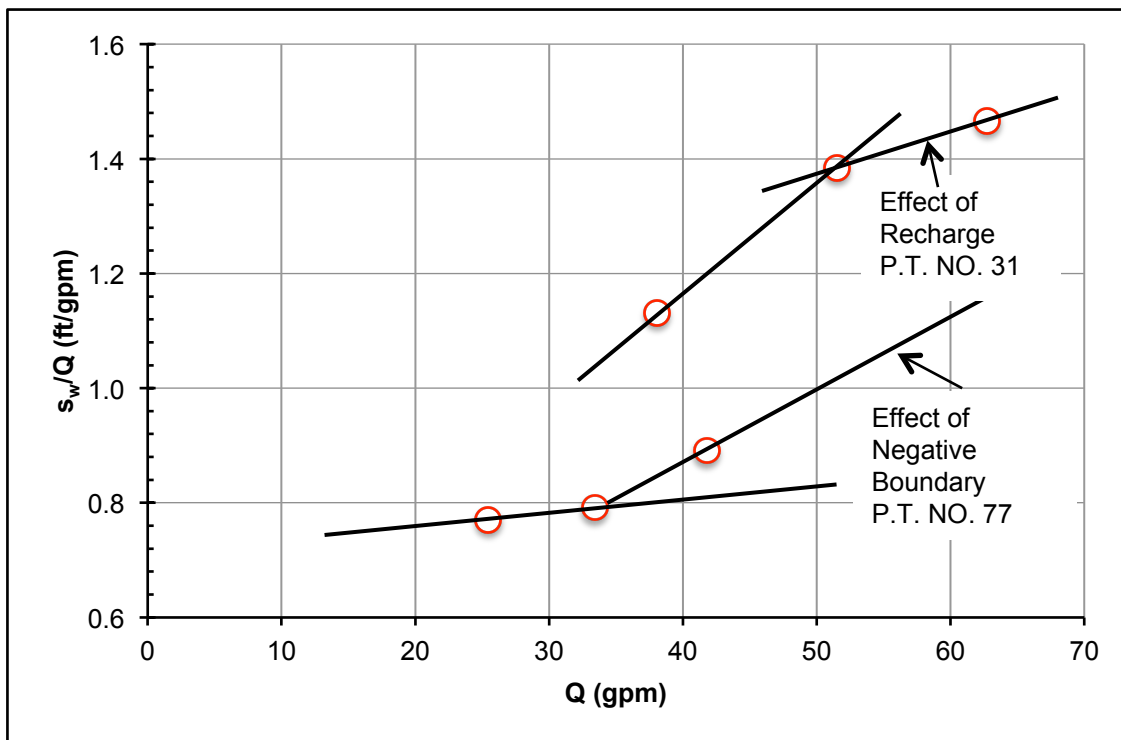
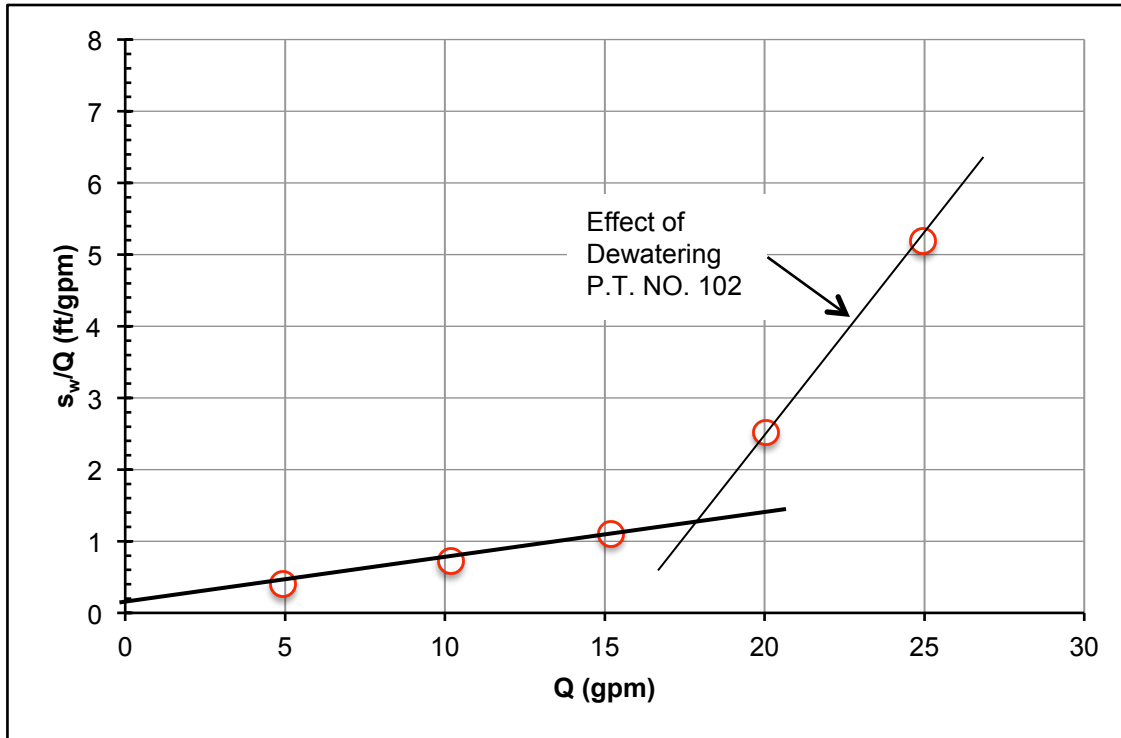
Figure 1.  
Semi-Logarithmic Plot of Drawdown vs. Time Elapsed



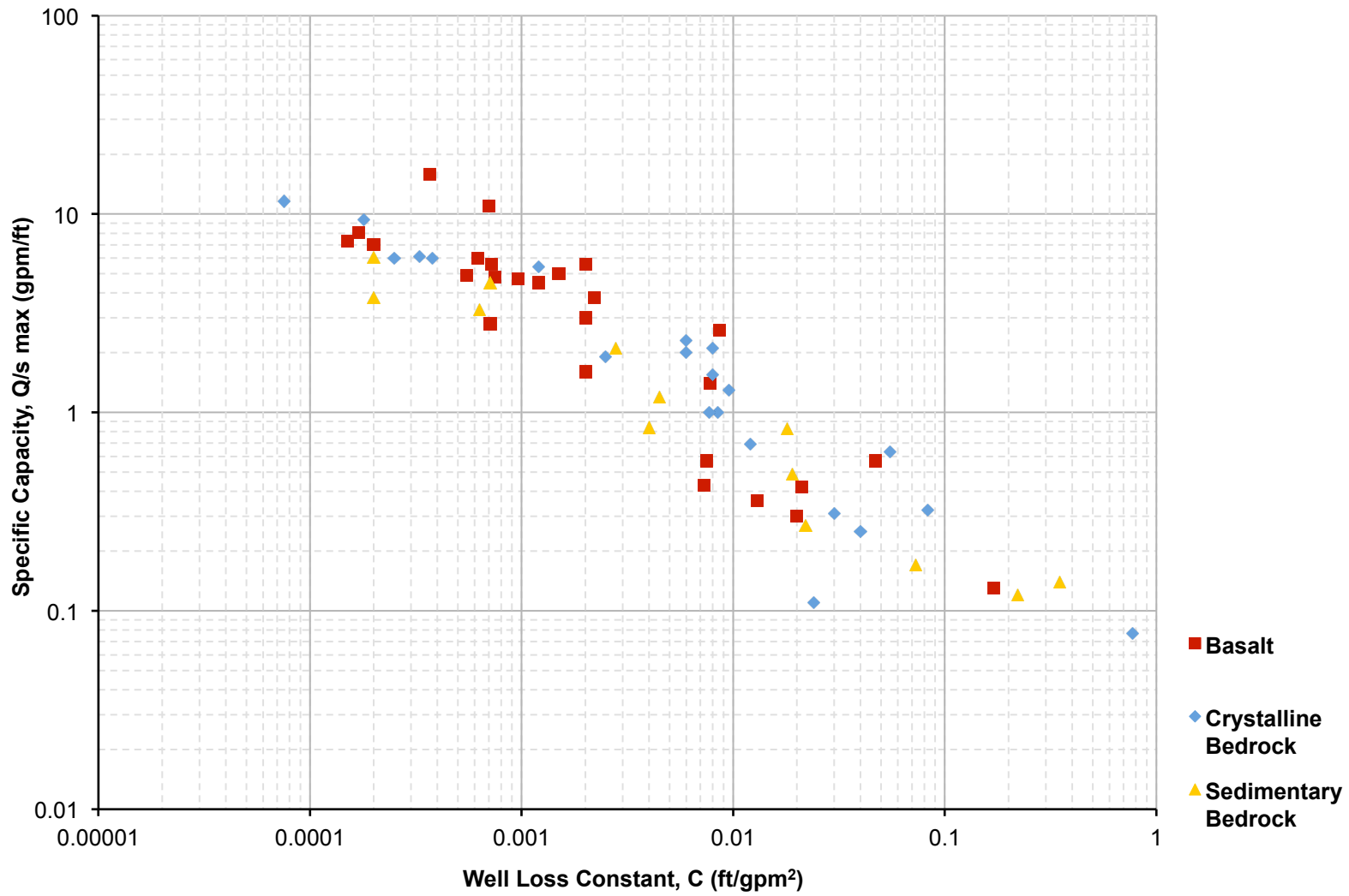
**Figure 2.**  
**Arithmetic Plot of  $s_w/Q$  vs.  $Q$**



**Figure 3: Plots of  $s_w/Q$  vs.  $Q$  Illustrating Fracture Dewatering, Leakage (Recharge) and Negative Boundary (Limited Aquifer) Conditions.**



**Figure 4.**  
**Specific Capacity (Q/s) vs. Well Loss Constant (C)**



**Figure 5**  
**Specific Capacity (Q/s) vs. Aquifer Loss Constant (B)**

